# Flight Mechanics Review

**a) General Rigid Body Dynamics Equations in 3D (12 Equations)**

A rigid body in 3D is fully described by 12 equations, split equally into 6 kinetic (dynamics) and 6 kinematic (geometric) equations.

**Kinetic Equations (Dynamics)**

These equations govern the forces and moments acting on the body.

**Translational Dynamics (Newton’s Second Law)**

**(1)**

**(2)**

**(3)**

*Where:*

* = Velocity components along the body x-, y-, and z-axes
* = External force components along the body axes
* = Mass of the body
* = Angular velocity components about the body x-, y-, and z-axes

**Rotational Dynamics (Euler’s Equations)**

**(4):**

**(5):**

**(6):**

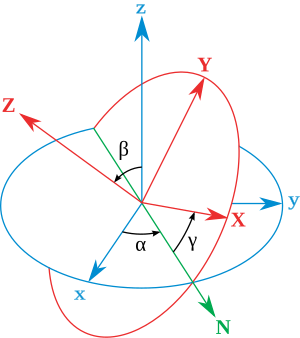
*Where:*

* = Moments of inertia about the body axes
* = External moment components about the body axes

**Kinematic Equations (Geometry)**

These equations relate the velocities and angular rates to the changes in position and orientation.

**Translational Kinematics (Position Updates)**

**(7):**

Euler angles

**(8):**

**(9):**

*Where:*

* = Inertial (earth-fixed) coordinates
* = Euler angles (roll, pitch, yaw)

**Rotational Kinematics (Euler Angle Rates)**

**(10):**

**(11):**

**(12):**

These 12 equations together comprehensively describe the motion of a rigid body in 3D space.

**b) Classification into Kinetics and Kinematics**

**Kinetic Equations (6 in total):**

* Equations (1) – (3): Translational dynamics
* Equations (4) – (6): Rotational dynamics

**Kinematic Equations (6 in total):**

* Equations (7) – (9): Relate body velocities (expressed in body frame) to inertial position changes
* Equations (10) – (12): Relate body angular rates to Euler angle rates

**c) Additional Equations for Fixed-Wing Airplane Equations of Motion (EOM)**

To specialize the general RBD equations for a fixed-wing airplane, the following are added: •

**Aerodynamic Force and Moment Models:**

Aerodynamic forces are expressed as functions of dynamic pressure, wing area, and non-dimensional coefficients (e.g., ). Similarly, aerodynamic moments (roll , pitch , yaw ) are modeled using coefficients that depend on angle of attack, sideslip, and control surface deflections.

* **Gravitational Force Projection:**

The weight of the airplane is resolved along the body axes via the transformation from the inertial (or earth-fixed) frame.

* **Velocity-to-Angle Relationships:**

Equations that relate the components of velocity in the body frame to the airspeed, angle of attack (), and sideslip angle ().

* **Control Input Effects:**

Additional relations that quantify how control surface deflections affect the aerodynamic forces and moments.

**d) Assumptions in Deriving Airplane Equations of Motion.**

Common assumptions include:

* **Rigid Body Approximation:** The aircraft is assumed not to deform.
* **Constant Mass and Inertia:** Effects of fuel burn or payload shifts are neglected.
* **Quasi-Steady Aerodynamics:** Aerodynamic forces and moments are assumed to respond instantaneously to changes in flight conditions.
* **Small Perturbations:** Linearization is often performed around a trim condition.
* **Neglection of High-Order Effects:** Compressibility, viscous effects, and higher-order nonlinearities are typically ignored in preliminary analyses.
* **Uniform Atmospheric Conditions:** The air is assumed to have steady and uniform properties.

**e) Mathematical Classification of the Airplane EOM.**

|  |  |
| --- | --- |
| Order | The equations are represented as first-order differential equations in state-space form. |
| Type | They are Ordinary Differential Equations (ODEs), with time being the independent variable. |
| Linearity | The full equations are nonlinear, although linearization is common near steady flight conditions. |
| Coupling | The equations are generally coupled, as translational and rotational dynamics interact. Under certain assumptions (such as decoupling of longitudinal and lateral-directional dynamics), the equations can be simplified to an uncoupled form. |

**f) Difference Between Body Axes and Earth (Inertial) Axes.**

* **Body Axes:**

A coordinate system fixed to the airplane, typically defined as:

* : along the fuselage (forward),
* : towards the right wing,
* : downward.

The body axes rotate with the aircraft.

* **Earth (Inertial) Axes:**

A fixed or quasi-fixed coordinate system relative to the Earth, often defined as North-East-Down (NED) or a similar scheme, serving as an absolute reference frame.

**g) Difference Between Pitch Angle () vs. Angle of Attack () and Sideslip Angle () vs. Heading Angle ().**

* **Pitch Angle () vs. Angle of Attack ():**

The pitch angle is the orientation of the aircraft’s longitudinal axis relative to the horizon, while the angle of attack is the angle between the chord line (or x-axis) and the oncoming airflow. In steady, coordinated flight these angles are related, but during maneuvers or in the presence of wind, they can differ significantly.

* **Sideslip Angle () vs. Heading Angle ():**

The sideslip angle is the angle between the aircraft’s longitudinal axis and the relative wind, whereas the heading angle is the navigational direction of the aircraft relative to a fixed reference (such as geographic North).

**h) Attitude Representations: Advantages and Disadvantages**

* **Euler Angles:**
* **Advantages**: Intuitive (roll, pitch, yaw) and straightforward.
* **Disadvantages**: Suffer from singularities (gimbal lock) when, for example, the pitch angle approaches
* **Direction Cosine Matrix (DCM):**
* Advantages: No singularity issues and provides a full transformation matrix.
* Disadvantages: Requires 9 elements along with orthogonality constraints, which can complicate numerical implementation.
* **Quaternions:**
* Advantages: Compact (4 parameters), computationally efficient, and free from singularities.
* Disadvantages: Less intuitive and involve a double-cover issue (i.e., q and −q represent the same orientation).

**• Axis-Angle Representation:**

* Advantages: Provides a clear geometric interpretation (rotation by a specific angle about a fixed axis).
* Disadvantages: Less practical for sequential rotations or for numerical integration when rotations are small.